ENP100 - Prosess og produksjon

Øving 10 - Løsningsforslag

Oppgave 1: Details are also given in Haugen (Dynamiske systemer) pp. 15 - 17
a)

$$\Delta p = \frac{1}{2}\rho\left(\frac{4 q_u}{\pi D^2}\right)^2 \cdot \xi(t) = \rho g h$$

Density cancels; Solve for q_u:

$$q_u^2 = \frac{2gh}{\xi(t)} \cdot \left(\frac{\pi D^2}{4}\right)^2 \Rightarrow q_u = \underbrace{\pi D^2 \sqrt{\frac{g}{8\,\xi(t)}}}_{K_v(t)} \cdot \sqrt{h}$$

[Units of K_v are $m^2 \cdot \sqrt{\frac{m}{s^2}} = \frac{m^2}{s}\sqrt{m}$]

By eqn. (2.6) in Haugen:

$$\frac{dh(t)}{dt} = \frac{1}{A}\left(q_i(t) - q_u(t)\right)$$

$$= \frac{q_i(t)}{A} - \frac{1}{A} \cdot K_v(t) \cdot \sqrt{h(t)} \qquad \underline{\text{QED}}$$

b)

- The output- or process variable "*y(t)*" is the liquid level level *h(t)*
- The disturbance - or non-controllable input variable "*v(t)*" is the inflow $q_i(t)$
- The control signal - or controllable input variable "*u(t)*" is the coefficient $K_v(t)$*

*: This coefficient *represents* the valve setting in the equation.

c)

$i = 1$

$t_1 = 1s, \quad \Delta t = t_1 - t_0 = 1s$

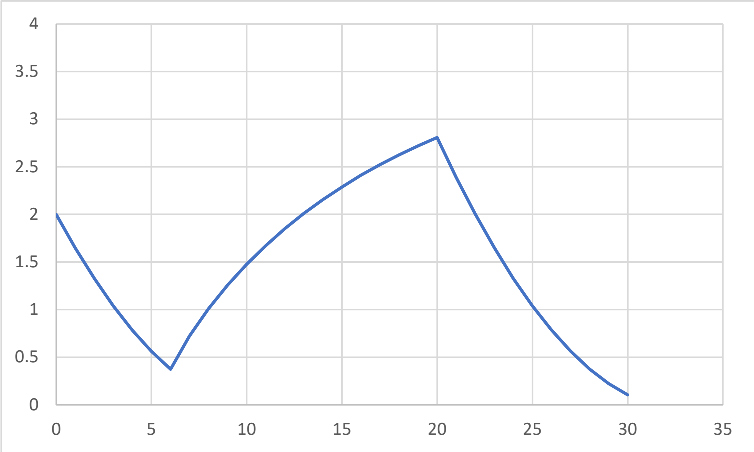
$$h(t_1) = 2m + \left(0 - \frac{1}{1m^2} \cdot 0.25\,\frac{m^2}{s}\sqrt{m} \cdot \sqrt{2m}\right) \cdot 1s$$

$$= 2 - 0.356 = \underline{\underline{1.646\ m}}$$

Oppgave 2:

See the excel sheet for details;

a) A plot of the level vs. time for the current settings should look like this:



The liquid just pours out until the inflow is "turned on" at t = 7 s. Then the level will start increasing, meaning that the inflow must be larger than the outflow at this point. As the level rises, however, the differential pressure across the valve will increase, and thereby also the outflow, resulting in that the *rate of level increase* becomes less with time.

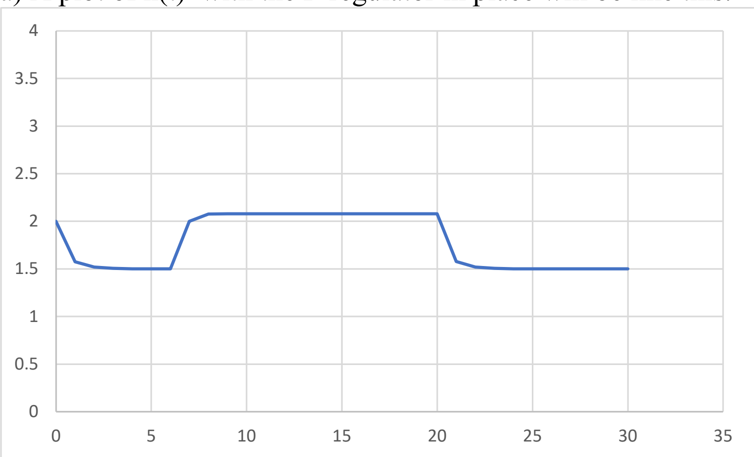
The maximum level, which is approximately 2.8 m is reached at t = 20 s, when the inflow is cut off.

After that the liquid flows out again, until the tank is empty.

b) By extending the time axis, a negative value for the next level will eventually be the case (occurs at t = 32 s), and with Excel this is reported as numerical error (#NUM!). The physical significance is of course h = 0, meaning "empty".

Optional; Putting in a P-regulator:

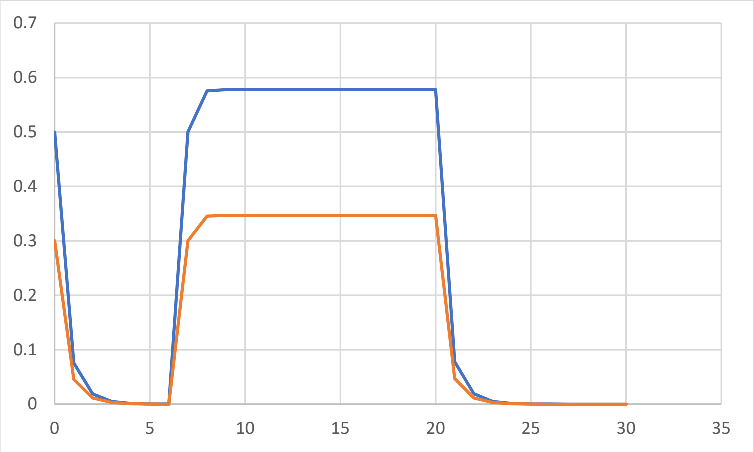
a) A plot of h(t) with the P-regulator in place will be like this:



The error is in this case defined so that a level higher than the desired one (the set point value) results in a positive value. Thus the higher the value of the error, the higher the value of u(t) = Kv(t). (the resistance coefficient in the denominator means that this must be lower, ergo a larger opening)

When the level approaches the set point from above, at some point the error is zero, and so is the control value, since it is proportional to the error. Kv = 0 means the valve is shut, so that the level cannot get below the set point.

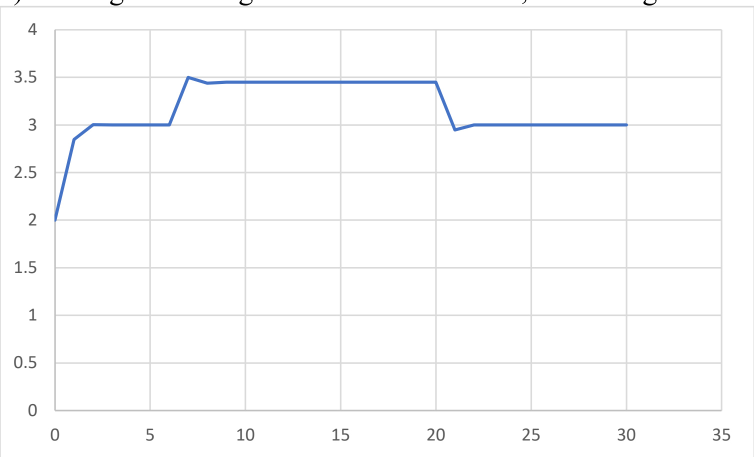
The plot below shows e(t) = h(t) - h_sp (blue) and u(t) = Kv (orange) vs. t. They have the same shape, i.e. are proportional to one another:



This is different when new liquid starts pouring in at t = 7. The error increases, and so does Kv, thereby quickly increasing the outflow, but once a stationary level (in this case a little above 2 m) is reached the value of u(t) = Kv will remain constant.

Using proportional action only will in general give a stationary error, i.e. the strategy is not succeeding in general.

b) Starting with a negative value of the error, something will seemingly start filling up the tank ...



Since the second term in the original differential equation (equation (2)) is the outflow divided by the area. Thereby a negative value of this term will physically signify pumping liquid in through the valve.

- Physically achievable with a pump or something, but not when adhering to the original premises.

Usually we will have to put certain restrictions into the equations when running simulations like this, in the current case this would have been done by replacing all negative values of u(t) with zero.