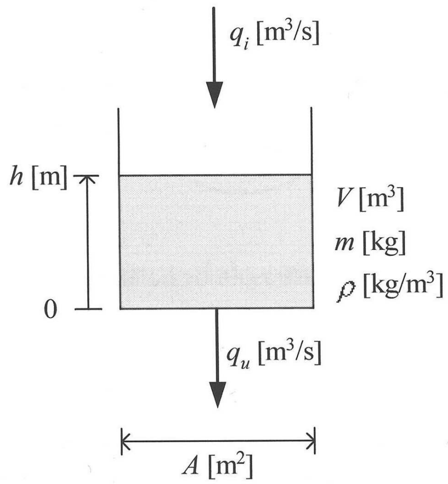
Øving 10 – 2022

Oppgave 1: Dynamic model

Figure 1 shows a tank of constant area, with liquid flowing in and out. The flow into the tank $q_i(t)$ is unpredictable, while the flow out $q_u(t)$ goes through a control valve with the following relation between differential pressure and flow:

$$\Delta p = \frac{1}{2}\,\rho \left(\frac{4\;q_u}{\pi\;D^2}\right)^2 \cdot \zeta \tag{1}$$



Figur 2.2: Væsketank

Figure 1: Liquid tank [1]

a) Suppose that the coefficient of resistance for the valve is adjustable $\big(\zeta = \zeta(t)\big)$. Show that that the differential equation for the liquid level h with respect to time can be written:

$$\frac{\mathrm{d}h(t)}{\mathrm{d}t} = \frac{q_i(t)}{A} - \frac{1}{A}\;K_v(t)\;\sqrt{h(t)} \tag{2}$$

b) Identify which of the variables in equation 2 playing the roles of output $y(t)$, disturbance $v(t)$, and control signal $u(t)$.

c) Equation 2 can be discretisized, and solved with respect to future values of the the liquid level h:

$$h(t_i) = h(t_{i-1}) + \left(\frac{q_i(t_{i-1})}{A} - \frac{1}{A}\;K_v(t_{i-1})\;\sqrt{h(t_{i-1})}\right) \cdot \Delta t \tag{3}$$

At $t_0 = 0$, the liquid level is 2 m. For a tank area $A = 1$ m^2, zero inflow ($q_i = 0$), and the coefficient K_v set to 0.25, what is the liquid level 1 second later ($\Delta t = 1$ s) ?

Oppgave 2: Simulation

In an Excel worksheet, set up 3 columns; one for time (t), one for inflow ($q_i(t)$) and one for the liquid level ($h(t)$). Also supply cells containing the area, and a constant value for K_v, see Figure 2.

Use $\Delta t = 1$ s, and let the time run from 0 to 30 s. In the interval $t = 7 - 20$ s, give the inflow $q_i(t)$ the value 0.5 m^3/s, otherwise it is zero.

Run a simulation to see how the level will vary with time;

a) What is the maximum liquid level in the time period, and when does it occur ?

b) What happens if the time is extended to, say, 35 s ?

P-regulator; (optional):

Add two additional columns, one for the error, $e(t) = h_{SP} - h(t)$, and one for the control signal, $u(t) = K_p \cdot e(t)$. Also supply cells holding values for the controller gain K_p and the set point h_{SP}.

Then replace the constant value of K_v in equation 3 with the control signal $u(t)$. This is the *feedback*, making the valve opening a function of the error. Use $K_v = 0.6$, and $h_{SP} = 1.5$ m.

a) Will this strategy succeed in maintaining the level at a desired value of 1.5 m ?

b) If the set point is altered to a level higher than the initial value of 2 m (say, $h_{SP} = 3$ m) the control signal $u(t)$ will attain negative values. What is the physical significance of this, and is it realistic ?

Literature

[1] Haugen, F.: *Dynamiske systemer, modellering, analyse og simulering*, 3. utgave, Fagbokforlaget, 2016, ISBN 978-82-519-2260-9

	A	B	C	D	E	F	G
1	A	1	m2		t	qi(t)	h(t)
2	Kv	0.25			0	0	2
3					1	0	=G2 + (F3/$B$1 - ($B$2/$B$1)*SQRT(G2))*(E3-E2)
4					2	0	
5					3	0	
6					4	0	
7					5	0	
8					6	0	
9					7	0.5	
10					8	0.5	
11					9	0.5	
12					10	0.5	
13					11	0.5	
14					12	0.5	
15					13	0.5	
16					14	0.5	
17					15	0.5	
18					16	0.5	
19					17	0.5	
20					18	0.5	
21					19	0.5	
22					20	0.5	
23					21	0	
24					22	0	
25					23	0	
26					24	0	
27					25	0	
28					26	0	
29					27	0	
30					28	0	
31					29	0	
32					30	0	
33							

Figure 2: Simulating tank level in Excel