MAF310 – Numerical modeling

Assignment 2 – Fall 2022

This assignment is due on 25th Nov.

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(1) Briefly explain what the following terms mean:

- Efficiency and stability in the context of root finding
- In what situations Newton-Raphson method is preferable to bisection method? When is bisection preferable to Newton-Raphson?
- How do you find roots of functions of many variables?
- (bonus) Explain briefly what is Ridder's method (you will need to open the book!) Contrast and compare it to Newton-Raphson and bisection

(2) The smallest positive, nonzero root of

$$\cosh x \cos x - 1 = 0$$

lies in the interval (4, 5). Find an approximate value of the root and give an estimate of the uncertainty of your answer.

(3) Explain briefly

- what are the assumptions going to approximating derivatives with finite difference approximations
- what is Richardson extrapolation

(4) Estimate $f'(2.37)$ from the following data

x	2.36	2.37	2.38	2.39
$f(x)$	0.5866	0.6289	0.6710	0.7129

Justify your method of choice.

(5) Briefly contrast and compare

- Romberg integration
- Gaussian quadrature

(6) Find approximate value of

$$\int_{-\infty}^{\infty} e^{-x^4/(1+x^2)}.$$

Give an estimate of the uncertainty of your answer. Justify your choice of method.

(7) Briefly explain in the context of ordinary differential equations

- Stiffness
- (bonus) Come up with a new example (= not in book) of stiff equations. Why are they stiff?
- Adaptive step size. When and why are they needed? How the step size can be judiciously chosen?

(8) Estimate $y(0.5)$, given the ordinary differential equation

$$y'(x) = \sin(y(x))$$

with the initial value $y(0) = 1$.

(9) Briefly explain the idea behind the two methods of solving boundary value problems

- Shooting method
- Finite difference method

(10) Find an approximate solution in the interval $0 \leq x \leq 4$ to

$$y''(x) = (2 + x)y(x), \qquad y(0) = 0 \qquad y'(1) = 4$$

Plot the solution. Justify your method of choice.