

MAF310 – Numerical modeling

Assignment 1 – Fall 2022

This assignment is due on Tuesday 27th Sept.

(1) Briefly explain what the following terms mean:

- Singular matrix and conditioning number
- Pivoting in the context of solving systems of linear algebraic equations
- Interpolation, extrapolation and curve fitting
- Spline
- Linear form in the context of curve fitting

(2) Systems of linear algebraic equations: Use Doolittle's decomposition to solve $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{pmatrix} -3 & 6 & -4 \\ 9 & -8 & 24 \\ -12 & 24 & -26 \end{pmatrix}, \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} -3 \\ 65 \\ -42 \end{pmatrix}.$$

(3) Interpolation: The points

$$\begin{array}{c|cccccc} x & -2 & 1 & 4 & -1 & 3 & -4 \\ \hline y & -1 & 2 & 59 & 4 & 24 & -53 \end{array}$$

lie on a polynomial. Use (for example) the divided difference table of Newton's method to determine the degree of the polynomial.

(4) Curve fitting: Determine a and b so that function $f(x) = axe^{bx}$ fits the following data in the least-squares sense. (Note that the function $f(x)$ is not quite just an exponential function!).

$$\begin{array}{c|ccccc} x & 0.5 & 1.0 & 1.5 & 2.0 & 2.5 \\ \hline y & 0.541 & 0.398 & 0.232 & 0.106 & 0.052 \end{array}$$